MAT226 Exam I: PRACTICE

The use of calculators is permitted on this exam; however, the correct answer without supporting work will not receive credit. No cell phones or other technology are allowed. Show all formulas used/work for full credit.

1. Given the universal set \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \) and the sets \( A = \{2, 4, 6, 8\} \) and \( B = \{0, 1, 2, 3, 4\} \), find the following sets and write the elements in the appropriate place in the Venn Diagram.

A. \( A \cup B = \)

B. \( A \cap B = \)

C. \( A - B = \)

D. \( \bar{A} \cap \bar{B} = \)

2. What is the formula for the \( n^{th} \) term of the given sequences?

A. 3, 7, 11, 15, 19, ...  \( a_n = \)

B. 3, 6, 12, 24, 48, ...  \( a_n = \)

C. 10, 50, 250, 1250, 6250, ...  \( a_n = \)

3. Write out the first 4 entries, \( (a_0, a_1, a_3, a_4) \) of the solution to the recurrence relations below with the given initial conditions.

A. \( a_n = na_{n-1} + 3, a_0 = 1 \)

B. \( a_n = n^2a_{n-1} - a_{n-2}, a_0 = 1, a_1 = 0 \)
4. A certain type of bacteria triples each hour if left untreated in the human stomach.
   A. If an initial population of 200 bacteria is introduced, write a recurrence relation which models this growth.

B. If a treatment with antibiotics kills 90% of the population over the course of the hour and a dose is taken each hour, write a recurrence relation which models this growth.

5. Solve the linear recurrence relation  \( a_n = 3a_{n-1} + 10a_{n-2} \), \( a_0 = 1, a_1 = 0 \)

6. Prove that the following equality is true for the values of \( n \geq 2 \). Show all work/explanations for full credit.

\[
\prod_{i=2}^{n} \left(1 - \frac{1}{i^2}\right) = \left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right) \cdots \left(1 - \frac{1}{n^2}\right) = \frac{n + 1}{2n}
\]
7. How many different 6 character license plates involving capital English letters and the digits 0-9 are there if
a) there are no restrictions?

b) there are three letters and three digits and the three letters appear together either at the beginning or end of the license?

8. In a computer cluster, often one “master” processor is in charge of assigning jobs to the other “slave” processors. In one such cluster, a master processor must assign 32 jobs to its slave processors. What is the minimum number of slave processors needed to ensure that no more than 6 jobs need to get assigned to any one processor?

9. A bagel shop sells 5 different types of bagels: plain, everything, salt, raisin, and blueberry. How many ways can a dozen (12) bagels be ordered if...
a) there are no restrictions.

b) you need to get at least one of each type.

c) you need at least 2 plain bagels and exactly 3 salt bagels.
10. Assume you have an unlimited supply of each type of the following fruits: apples, oranges, bananas, and pears. How many ways can you select 10 pieces of fruit?

11. If there are 5 faculty and 12 students in a small math department and you want to form a research group with 3 faculty and 6 students. How many ways can this be done...
   a) with no other restrictions?

   b) if two of the students either want to be in the research group together or not at all?

12. A group of eight adults and twelve children arrive at a restaurant and ask that the adults be sat at a separate table from the children. The 8 adults are sat at a round table and the 12 children are sat at a long rectangular table. How many different seating arrangements are possible if rotations around the round table are considered the same and reflections about either center line of the rectangular table are considered the same? Draw some pictures to help decide about the reflections. Feel free to ask for clarification if you need it on this one.