Charged Particle (Chip) Model for Integer Multiplication and Division

Apply and extend previous understandings of operations with fractions.

CCSS.Math.Content.7.NS.A.2
Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

CCSS.Math.Content.7.NS.A.2.a
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as \((-1)(-1) = 1\) and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

CCSS.Math.Content.7.NS.A.2.b
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If \(p\) and \(q\) are integers, then \(-p/q = (-p)/q = p/(-q)\). Interpret quotients of rational numbers by describing real-world contexts.

Exploration # 1: Integer Multiplication

Use the “Repeated Addition” Model for Multiplication to solve the following problems:

\[3 \times 5\]

Represent three sets of 5 with 5 yellow chips in three sets.

Take the union of the three disjoint sets and you have 15 yellow chips = 15.

a) \(4 \times 3\)
b) \(3 \times (-6)\)

c) \((-7) \times 3\)

Example: \((-2) \times (-3)\). Interpretation: “Take away two sets of negative three”.

Start with zero. You want to take away one set of negative three, but you have no chips.

In order to take away one set of negative three, you add in three “zeros” = 3 yellow and 3 red chips. Take away one set of negative three (three red chips).

In order to take away a second set of negative three, you add in three “zeros” again and take away the second set of negative three (three red chips).

You are left with 6 yellow chips = 6.
Exploration # 2: Integer Division

Use the “Partition” Model for Division (= Division by Sharing) to solve the following problems:

a) \(12 \div 4\)

\[
\begin{align*}
d) & \ (-3) \times (-4)
\end{align*}
\]

\[
\begin{align*}
\text{a) } & \ 12 \div 4 \\
\text{b) } & \ (-6) \div 2 \\
\text{c) } & \ (-5) \div 2
\end{align*}
\]

Represent \(-15\) with 15 red chips.

Partition 15 into three sets. Each set will have 5 red chips = -5.
Use the “Repeated Subtraction” Model for Division to solve the following problems:

-10 ÷ -5

Represent -10 with 10 red chips.

How many sets of -5 can we subtract from -10?
You can subtract (take away) two sets of 5 red chips.
-10 ÷ -5 = 2

c) (-12) ÷ (-3)

d) 20 ÷ 5