A BRIEF INTRODUCTION TO

MULTIVARIATE ANALYSIS OF VARIANCE (MANOVA)

Like the analysis of variance (ANOVA), the multivariate analysis of variance (MANOVA) has variations. For example, the one-way MANOVA contains a single factor (independent variable) distinguishing participants into groups and two or more quantitative dependent variables. For example, one could do three separate one-way ANOVAs; however, using MANOVA, you will see how the combination of the three variables distinguishes the groups, in one analysis. There is a two-way or two-factor MANOVA that has two independent variables and two or more quantitative dependent variables. A doubly multivariate or mixed MANOVA has a between groups independent variable and a repeated measures (within groups) independent variable and two or more quantitative dependent variables. Mixed MANOVAs are one way to analyze intervention (experimental) studies that have more than one dependent variable. The MANCOVA (Multivariate Analysis of Covariance) is used when you include both categorical/discrete (independent variable) and continuous (covariate) variables as predictors of the linear combination of two or more quantitative dependent variables.

As with ANOVA, the independent variables for a MANOVA are factors, and each factor has two or more levels. Unlike ANOVA, MANOVA includes multiple dependent variables rather than a single dependent variable. MANOVA evaluates whether the population means on a set of dependent variables vary across the levels of a factor or factors. That is, a one-way MANOVA tests the hypothesis that the population means for the dependent variables are the same for all levels of a factor (across all groups). If the population means of the dependent variables are equal for all groups, the population means for any linear combination of these dependent variables are also equal for all groups. Consequently, a one-way MANOVA evaluates a hypothesis that includes not only equality among groups on the dependent variable, but also equality among groups on linear combinations of these dependent variables.

The multivariate analysis of variance (MANOVA) is a complex statistic similar to ANOVA but with multiple dependent variables analyzed together. That is, the MANOVA is a multivariate extension of ANOVA. The dependent variables should be related conceptually, and they should be correlated with one another at a low to moderate level. If they are highly correlated, one runs the risk of multicollinearity. If they are uncorrelated, there is usually no reason to analyze them together. The General Linear Model program in SPSS provides you with a multivariate *F* based on the linear combination of dependent variables that maximally distinguishes your groups. This multivariate result is the MANOVA. SPSS also automatically prints out univariate *F*s for the separate univariate ANOVAs for each dependent variable. Typically, these ANOVA results are not examined unless the multivariate results (the MANOVA) are significant, and some statisticians believe that they should not be used at all.

SPSS reports a number of statistics to evaluate the MANOVA hypothesis, labeled *Wilks' Lambda*, *Pillai's Trace*, *Hotelling's Trace* (*T*), and *Roy's Largest Root*. Each statistic evaluates a multivariate hypothesis that the population means on the multiple dependent variables are equal across groups. Most statisticians use *Wilks' Lambda, Λ*, and as such, it is frequently reported in the social (and behavioral) sciences literature. *Pillai's Trace* is a reasonable alternative to *Wilks' Lambda* for use with the MANOVA. Measures of effect size are often reported with (partial) eta squared (η^2 is reported in SPSS) or eta (η is the square root of eta squared). A multivariate effect size index is more commonly used (see *Multivariate η^2* below).

If the one-way MANOVA is significant, follow-up analyses can assess whether there are differences among groups on the population means for certain dependent variables and for particular linear combinations of dependent variables. A popular follow-up approach is to conduct multiple ANOVAs, one for each dependent variable, and to control for Type I error across these multiple tests using one of the Bonferroni approaches (e.g., α / number of dependent variables). If any of these ANOVAs yield significance and the factor contains more than two levels (with two levels, a comparison of group means is conducted), additional follow-up tests are performed. These tests typically involve post hoc pairwise comparisons among levels of the factor, although they may involve more complex comparisons. For example, some statisticians prefer conducting parameter estimates, with the use of dummy variables devised to distinguish one group from the others. Parameter estimates tell us how the dependent variables are weighted in the equation that maximally distinguishes the groups.

Some statisticians have criticized the strategy of conducting follow-up ANOVAs after obtaining a significant MANOVA because the individual ANOVAs do not take into account the multivariate nature of the MANOVA. Conducting follow-up ANOVAs ignores the fact that the MANOVA hypothesis includes sub-hypotheses about linear combinations of dependent variables. Of course, if we have particular linear combinations of variables of interest, we can evaluate these linear combinations using ANOVA in addition to, or in place of, the ANOVAs conducted on the individual dependent variables. For example, if two of the dependent variables for a MANOVA measure the same construct of introversion, then we may wish to represent them by transforming the variables to z scores, adding them together, and evaluating the resulting combined scores using ANOVA. This ANOVA could be performed in addition to the ANOVAs on the remaining dependent variables. If we have no clue as to what linear combinations of dependent variables to evaluate, we may choose to conduct follow-up analyses to a significant MANOVA using discriminant analysis. Discriminant analysis yields one or more uncorrelated linear combinations of dependent variables that maximize differences among groups. These linear combinations are empirically determined and may not be interpretable.

Assumptions underlying the One-Way MANOVA

Assumption 1: The dependent variables are multivariately normally distributed for each population, with the different populations being defined by the levels of the factor. This is commonly referred to as the *assumption of multivariate normality*.

If the dependent variables are multivariately normally distributed, each variable is normally distributed ignoring the other variables and each variable is normally distributed at every combination of values of the other variables. It is difficult to imagine that this assumption could be met. To the extent that population distributions are not multivariately normal and sample sizes are small, the p values may be invalid. In addition, the power of the MANOVA test may be reduced considerably if the population distributions are not multivariately normal and, more specifically, thick-tailed or heavily skewed. The Shaprio-Wilks test can be used to test univariate normality for each dependent variable, which in turn gives insight to the multivariate normality assumption (as univariate normality is a necessary condition of multivariate normality).

Assumption 2: The population variance and covariances among the dependent variables are the same across all levels of the factor. That is, variances for each dependent variable are approximately equal in all groups plus covariances between pairs of dependent variables are approximately equal for all groups. This is commonly referred to as the *assumption of homogeneity of variance-covariances matrices*.

To the extent that the sample sizes are disparate and the variances and covariances are unequal, MANOVA yields invalid results. SPSS allows us to test the assumption of homogeneity of the variance-covariance matrices with *Box's M* statistic. The *F* test from *Box's M* statistics should be interpreted cautiously in that a significant result may be due to violation of the multivariate normality assumption for the *Box's M* test, and a non-significant result may be due to a lack of power. It is advised to use $p < .001$ as the criterion for testing significance.

Assumption 3: The participants are randomly sampled, and the score on a variable for any one participant is independent from the scores of this variable for all other participants. That is, each person's scores are independent of every other person's scores. This is commonly referred to as the *assumption of independence.*

MANOVA should not be conducted if the independence assumption is violated.

MANOVA is robust to violations of multivariate normality and to violations of homogeneity of variance-covariance matrices if groups are of nearly equal size (*N* of the largest group is no more than 1.5 times the *N* of the smallest group).

Effect Size statistics for a One-Way MANOVA

The multivariate General Linear Model procedure (in SPSS) computes a multivariate effect size index. The multivariate effect size associated with *Wilks' Lambda* (*Λ*) is the multivariate eta square:

$$\text{Multivariate } \eta^2 = 1 - \Lambda^{\frac{1}{s}}$$

Here, *s* is equal to the number of levels of the factor minus 1 or the number of dependent variables, whichever is smaller. This statistic should be interpreted similar to a univariate eta square and ranges from 0 to 1. A 0 indicates no relationship between the factor and the dependent variable, while a 1 indicates the strongest possible relationship. It is unclear what should be considered a small, medium, and large effect size for this statistic. As such, the literature should be consulted to assist in its interpretation relative to the discipline (field) of study for which the MANOVA is being used.

References

Green, S. B., & Salkind, N. J. (2003). Using SPSS for Windows and Macintosh: Analyzing and Understanding Data (3rd ed.). Upper Saddle River, NJ: Prentice Hall.

Leech, N. L., Barrett, K. C., & Morgan, G. A. (2005). SPSS for Intermediate Statistics: Use and Interpretation (2nd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.