MAT 667 (Dynamical Systems), Prof. Swift
Ideas for projects

The projects are meant to be a fairly small explorations. I want you to give a 15-20 minute talk about your project to the class. You may work in groups of 2 or 3. I will ask you to talk twice. The second talk can be an update on additional progress, or it can be a new project.

You do not need to turn in a written paper in addition to the talk, unless you do a "chalk talk". Please send me email atachments of your power point or beamer slides, along with any computer programs (if applicable).

Here are some ideas about projects: Please feel free to talk to me about any idea that interests you.

1. Explore the three dimensional parameter space for the driven damped pendulum,

$$\frac{d^2u}{dt^2} = -\sin(u) - c\frac{du}{dt} + \rho\cos(\omega t).$$

or the driven Duffing equation,

$$\frac{d^2u}{dt^2} = u - u^3 - c\frac{du}{dt} + F\cos(\omega t).$$

Use some numerical integration (for example the Mathematica notbook on the web site) to find solutions. Eliminate transients and look for stable periodic orbits, chaotic orbits, period doubling bifurcations, and multiple stable solutions at the same parameter value. The result will be a "map" of the 3-dimensional parameter space. Fix one parameter fixed and then plot other 2 parameters in the plane, indicating the regions in that plane with the various behaviours. Then do it for another valued of the fixed parameter.

2. Do an analysis of the parameter space available in the Chaos Machine (or driven washboard) that was brought to class. Since the shape of the washboard is fixed, and the the only parameters we can vary are the friction constant c and the frequency ω_d. The machine has $L = 3$ inches and $H = 1/8$ inch in the notation of Problem 3 in Homework 1. Do numerical simulations of the system, with $A = 0.9$ inches, and the driving period, $2\pi/\omega_d$ varies from about 0.66 to 1.6 seconds. See if you can find a value of c that shows sequence of a period doublings.

3. Modify the DrivenDampedPendulum.nb notebook from class to use Newton's method to find (possibly unstable) fixed points of the Poincaré map . Let P be the Poincaré map, that takes $(\theta, \dot\theta)$ at $t = 0$ to these values at $t = 2\pi/\omega$. Figure out how to compute the derivative DP, and do Newton's method to solve $P(\theta, \dot\theta) - (\theta, \dot\theta) = (0, 0)$ for period 1 orbits of the Poincaré map, or $P^{(k)}(\theta, \dot\theta) - (\theta, \dot\theta) = (0, 0)$ for period k (or a divisor of k) orbits. You may also want to solve $P(\theta, \dot\theta) - (\theta + 2\pi, \dot\theta) = (0, 0)$ to find rolling period 1 orbits that travel one spatial period to the right.

4. Learn about the Lagranian method in classical mechanics. This uses "generalized coordinates" so you do not have to compute components of forces like in usual Newtonian machanics. This can be applied to several situations. You can choose one or more of these, or some other application.

- A ball rolling on a surface $z = f(x)$, or a bead sliding on a wire with that shape. Contrast the dynamics with the approximate $\ddot{x} = -f'(x)$ obtained when the slope $f'(x)$ is small, in some scaled sense.
- Investigate the "inverse problem" from the previous one. That is, what shape of surface $z = f(x)$ is needed to get a given force function $F(x)$ with dynamics $\ddot{x} = F(x)$. If $F(x) = -kx$, so the period of oscillations is independent of amplitude, this is the famous tautochrone problem, and the shape $y = f(x)$ is a catenoid.
- Investigate the equations of motion of a chanrged particle in an electric and Magnetic field.
- Investigate the equations of motion in a rotating coordinate system

5. Do a literature search, and/or a numberical investigation of the restricted three-body problem of classical mechanics. Two bodies of Mass M and masss m rotate in a circular orbit about their center of mass. A third body of negligable mass moves in their plane of motion. This can be reduced to a two-degree of freedom Hamiltonian system.

6. Write an applet that does not use java, and hence is considered "safe" on a web site, to do a demonstration of a dynamical system. Bonus points if you can include the "exclusive or" printing or the sound mentioned by me in class.

7. The Alligood textbook has many "Challenges" that would be good pencil-and-paper projects.

8. Do a report on iterated maps on the complex plane. In particular, report on Jula sets and the Mandelbrot set.

9. Write a program that will plot a histogram of density for the logistic map as a function of the parameter a. Compute the Lyapunov exponent to get a heuristic for how transient iterates to do before collecting data for the histogram. Then, make a movie showing how this histogram changes as a function of the parameter a.

This can be done in Mathematica or MATLAB, but a compiled language would be faster, especially when making the movie.

10. Design a machine that would use the "shaken washboard" demonstrated in class as a base for a driven Duffing oscillator: $\ddot{x} + c\dot{x} - x + x^3 = F\cos(\omega t)$. A frame with the shape $y = -\frac{k}{2}x^2 + \frac{\ell}{4}x^4$ can be put on top of the sinusoidal metal bar. The 3D printers available at Cline library might be able to make this. Investigate the values of k and ℓ, and the ball-frame interface (friction) so that the machine would show period doubling, as well as chotic motion with random traverses from one well to the other. The bar moves back and forth with position $A\cos(\omega t)$, where $a = 0.9$ inches, with a period $2\pi/\omega$ from about 0.66 to 1.6 seconds.

11. Write a program that computes the Poincaré map for the Lorenz equations with the standard parameters. Use this to get a very long sequence of -1s and 1s, representing L's and R's for an orbit in the attractor starting from some random initial condition. Test the null hypothesis that this is the result of an iid coin flip. My guess is that the null hypothesis is false because there are too many long strings of the same letter.

Then process the sequence to get a new sequence of the number of copies of the same letter in a row. For example RRRLLRRRLRLRRRLLLR gets mapped to 2311133. (Do not use the initial RRR and the final R. We do not know how many R's preceed the initial R.) Test the hypothesis that this new sequence is iid.

You can get many "realizations" of the sequences by starting with different random initial conditions, or by chopping up a very long sequence into sub-sequences.

You may do any type of statistical analysis on these sequences that makes sense. Don't be constrained by what I suggested here. I have audited STA 570, but taken no other statistics classes. So a big part of this project is to understadwhat I'm trying to say here and restate it with proper statistical jargon.

12. Do a report on the work of Wawick Tucker, entitled "The Lorenz attractor exists." In the late 1990's he did a rigorous proof Lorenz's 1963 conjecure. You can search for "Lorenz attractor exists" to find his papers on the web. A good place to start on this is with the review article "Mathematics: The Lorenz attractor exists", Nature 406, 948-949 (31 August 2000), by Ian Stewart. You can get this via the Cline library web site.