The Trapezoid Method is Exact For Linear functions

Example: Suppose f is a linear function, and $f(-2)=7$ and $f(4)=-3$. Evaluate $\int_{-2}^{4} f(x)\,dx$ with 2 methods: (1) Using The Trapezoid Rule (2) Using antiderivatives

(1) $\int_{-2}^{4} f(x)\,dx = \frac{(4+2)}{2}\left[f(4)+f(-2)\right] = \frac{6}{2}\left[-3+7\right] = \frac{6\cdot 4}{2} = \boxed{12}$

(2) $f(x) = 7 + \frac{(-3-7)}{4+2}\cdot(x+2) = 7 - \frac{10}{6}(x+2) = 7 - \frac{5}{3}(x+2)$

$= 7 - \frac{5}{3}x - \frac{10}{3} = -\frac{5}{3}x + \frac{11}{3}$

$\int_{-2}^{4}\left(-\frac{5}{3}x + \frac{11}{3}\right)dx = \left(-\frac{5x^2}{3\cdot 2} + \frac{11}{3}x\right)\Big|_{-2}^{4} = \frac{-5\cdot 4^2}{3\cdot 2} + \frac{11\cdot 4}{3} - \left(\frac{-5(-2)^2}{3\cdot 2} + \frac{11}{3}(-2)\right)$

Error in class was here

$= \frac{-5}{6}(16-4) + \frac{11}{3}(4-(-2)) = \frac{-5}{6}\cdot 12 + \frac{11\cdot 6}{3}$

$= \frac{-5\cdot 6}{3} + \frac{11\cdot 6}{3} = -10 + 22 = \boxed{12}$