MAT 136 - Swift

Use the definition of the derivative to show that $f'(x) = 2x+3$, for the function $f(x) = x^2 + 3x + 1$

$\boxed{\text{Method 1}}$. Step 1: Simplify the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 3(x+h) + 1 - (x^2 + 3x + 1)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + \boxed{3x} + 3h + 1 - x^2 \boxed{-3x} - 1}{h}$$

$$= \frac{2xh + h^2 + 3h}{h}$$

$$= \frac{h(2x + h + 3)}{h}$$

$$= 2x + h + 3, \quad \text{provided } h \neq 0$$

Step 2: Use def.

Therefore, $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ using Step 1.

$$= \lim_{h \to 0} (2x + h + 3) = 2x + 0 + 3$$

so $\boxed{f'(x) = 2x + 3}$

$\boxed{\text{Method 2}}$ Start with the definition.

Write "lim" every time!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) + 1 - (x^2+3x+1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h + 1 - x^2 - 3x - 1}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h+3)}{h}$$

The magic step.

$$= \lim_{h \to 0} (2x+h+3)$$

$$= 2x + 0 + 3$$

$$\boxed{f'(x) = 2x+3}$$