MAT 136 Group Work on Continuity

Break into groups. Each group will get an extra copy of this sheet to write the answers on. Turn in *one sheet for each group of 3 or 4* and keep one for yourself. This group work is before the end of class today, and is worth 5 class points.

Names: _____ _____ _____ _____

Definition: A function f is **continuous at a number** c if and only if $\lim_{x\to c} f(x) = f(c)$.

We can also say that "f is continuous at c" or "f is continuous at $x = c$."

On thing we know from this definition is

If f is continuous at 3 then $\lim_{x\to 3} f(x) = f(3)$.

This is in fact the way we compute most limits by hand. Almost any function you can plot with your calculator is continuous at every number in its domain. To compute $\lim_{x\to c} f(x)$ when $f(c)$ is not defined, we algebraically simplify f to get an expression that agrees with f for all $x \neq c$ but *is* defined at $x = c$.

Today we are going to talk about the other direction in the definition of continuity:

If $\lim_{x\to c} f(x) = f(c)$ then f is continuous at c

The one statement $\lim_{x\to c} f(x) = f(c)$ is equivalent to these four statements:

1. $f(c)$ is defined.
2. $\lim_{x\to c^-} f(x)$ exists.
3. $\lim_{x\to c^+} f(x)$ exists.
4. All three of these numbers are equal. That is $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$.

1. Find the values of a and b such that the following function f is continuous at 3. Explain your reasoning. After you found a and b, sketch a graph of f with these parameters.

$$f(x) = \begin{cases} ax & \text{if} \quad x < 3 \\ b & \text{if} \quad x = 3 \\ 2 - x & \text{if} \quad x > 3 \end{cases}$$

One of the only functions you can plot with your calculator that is *not* continuous on its domain is the floor function, or greatest integer function, defined by

$$\lfloor x \rfloor = \text{the largest integer less than or equal to } x.$$

In other words, round x down to get $\lfloor x \rfloor$. For example $\lfloor 4 \rfloor = 4$, $\lfloor 1.5 \rfloor = 1$, and $\lfloor -1.5 \rfloor = -2$. TI calculators use `int(x)` for the floor function. To get to `int(x)` on the TI-83, press the MATH button, then go right to NUM, and `int(x)` is number 5. Plot with `Zoom Decimal`.

2. Show, using the definition of continuity, that $f(x) = \lfloor x \rfloor$ is continuous at $x = 1.5$.

3. Show, using the definition of continuity, that $f(x) = \lfloor x \rfloor$ is not continuous at $x = 2$.

4. Write an English sentence describing the numbers where $f(x) = \lfloor x \rfloor$ is continuous, and where f is not continuous.

5. Is $g(x) = \dfrac{1}{x}$ continuous on its domain? Is g continuous at all real numbers? Explain, and include a sketch.

6. Was there a time in your life when your height in inches was equal to your weight in pounds?