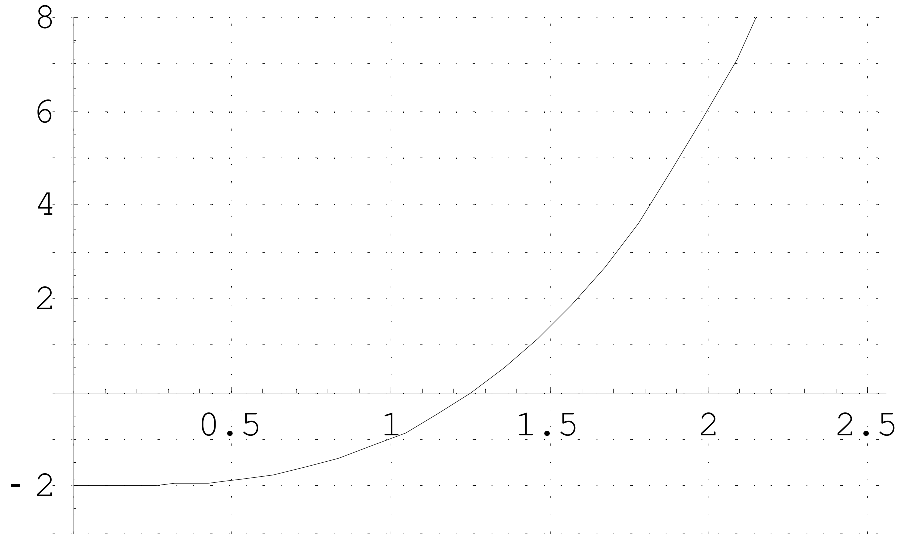
MAT 136, Prof. Swift
In-Class Worksheet on Newton's Method

Newton's method is a very efficient algorithm for solving $f(x) = 0$. Of course, you can do this with your graphing calculator using the "Calculate zero" or "Calculate root" feature. The calculator uses something like Newton's method. The purpose of this exercise is to understand how Newton's method works, both graphically and with formulas. You will also learn how to do Newton's method quickly on your calculator.

As an example, let's solve $x^3 - 2 = 0$ by a graphical Newton's method. (Note that the solution is the cube root of 2, which we can easily get with our calculator. In WeBWorK you will use Newton's method to solve equations like $20\sin(x) = x$, which do not have closed-form exact solutions.) We'll do it together in class, starting with the initial guess $x_1 = 2$.



Now try to repeat the first graphical step with formulas:
(Do this!) Starting with $x_1 = 2$, and using the formula $f(x) = x^3 - 2$, find the equation for the tangent line to the curve at the point $(2, f(2)) = (2,6)$.

(Do this!) Find the x-intercept of the tangent line you just found. This number is x_2.

(Do this!) If you have a general function $f(x)$, find an equation for the tangent line at the point $(x_1, f(x_1))$. Then find the x-intercept of the tangent line. This number is x_2. Get a formula for x_2 in terms of x_1, $f(x_1)$ and $f'(x_1)$.

Newton's method is an iterative process: You start with an initial guess (x_1) and get approximations x_2, x_3, x_4, etc. using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Let us do Newton's method on our calculators together to solve $x^3 - 2 = 0$ with the initial guess $x_1 = 2$. The general formula for Newton's method, with $f(x) = x^3 - 2$, becomes

$$x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2}$$

For example, we can compute x_2 without a calculator:

$$x_2 = x_1 - \frac{x_1^3 - 2}{3x_1^2} = 2 - \frac{2^3 - 2}{3 \cdot 2^2} = 2 - \frac{6}{12} = 1.5$$

We would probably want to use a calculator to compute x_3:

$$x_3 = x_2 - \frac{x_2^3 - 2}{3x_2^2} = 2 - \frac{1.5^3 - 2}{3 \cdot 1.5^2} = 1.296296296...$$

Now the real problem comes. Do we want to punch in that ugly decimal 3 times to compute x_4? Of course not!

Here's how to avoid excessive number punching, using the store button "STO". On the TI-83 it is the button above the bottom left button.

Initialize: Store the initial guess into the X variable:
2 STO X (So now X is $x_1 = 2$.)

Update X to be the next guess:
X – (X^3-2)/(3*X^2) STO X (Now X is $x_2 = 1.5$.)

Repeat that same command again. You don't have to punch it in though, just press ENTER (Now X is $x_3 = 1.296296296...$ and that number should appear on your screen.)

Continue pressing ENTER until the numbers don't change. In other works, stop when $x_{n+1} = x_n$.) In this example, a few ENTERs gave me 1.25992105 which kept repeating when I pressed ENTER again. This repeating number is the numerical approximation to a solution of $x^3 - 2 = 0$.