This is the exam I gave last year. The solutions are on the web. You can get there by following the link on our “Exams” page.

Be prepared to give a code name for posting results on web.

**Groundrules:** No notes are allowed. A graphing calculator is expected.

If the problem has a “W”, show your work! If you don’t show your work, or you have errors in the work, I may take off points even if you get the correct answer.

1. (W) Find the domain of each of the following functions. (Use interval notation, with “∪” for union if necessary.)
   
   (a) \( f(x) = \sqrt{8 + 7x - x^2} \)
   
   (b) \( g(x) = \frac{1}{\sqrt{10x + 15}} \)
   
   (c) \( h(x) = \log_a(x^2 - 25) \)

2. (a) Find an expression for the linear function \( f \) with values \( f(20) = 35 \) and \( f(23) = 36 \). I suggest you use the form \( f(x) = m(x - x_0) + y_0 \).

   (b) Find an expression for the exponential function \( g \) with values \( g(20) = 35 \) and \( g(23) = 36 \). I suggest you use the form \( g(x) = y_0a^{(x-x_0)/h} \).

3. (W) Given that \( f(x) = x^2 - 8 \) and \( g(x) = \sqrt{x + 7} \), calculate

   (a) \( f \circ g(x) = \), its domain is ________

   (b) \( g \circ f(x) = \), its domain is ________

   (c) \( g \circ g(x) = \), its domain is ________

4. Starting with the graph of \( f(x) = 3^x \), write an equation of the graph that results from

   (a) Shifting \( f(x) \) 2 units to the right. \( y = \) __________

   (b) Shifting \( f(x) \) 4 units upward. \( y = \) __________

   (c) Reflecting \( f(x) \) about the x-axis. \( y = \) __________
5. If \( \log p = x \) and \( \log q = y \), evaluate the following in terms of \( x \) and \( y \):

(a) \( \log(p^{-8}q^3) = \)

(b) \( \log \sqrt{p^2q^{-5}} = \)

(c) \( (\log p)^{-3} = \)

6. The figure on the left is the graph of a function \( f \). Recall that a dot means that
the point is on the graph, and an open circle means that the point is not on the graph.

(a) Sketch the graph of \( f^{-1} \) on the right.

(b) What is the domain of \( f \)? (Use interval notation, with “∪” for union if necessary.)

(c) What is the domain of \( f^{-1} \)?

7. Consider the same function \( f \) whose graph is shown in the previous problem. Fill
in the blanks with a real number, “undefined” if the function is undefined, or “DNE”
if a limit does not exist.

(a) \( f(1) = \)

(b) \( f(1.5) = \)

(c) \( f(2) = \)

(d) \( f^{-1}(1) = \)

(e) \( f^{-1}(1.5) = \)

(f) \( f^{-1}(2) = \)

(g) \( \lim_{x \to 1^-} f(x) = \)

(h) \( \lim_{x \to 1^+} f(x) = \)

(i) \( \lim_{x \to 1} f(x) = \)

(j) \( \lim_{x \to 1.5^-} f(x) = \)

(k) \( \lim_{x \to 1.5^+} f(x) = \)

(l) \( \lim_{x \to 1.5} f(x) = \)
8. The ellipse \[ \frac{x^2}{3^2} + \frac{y^2}{5^2} = 1 \]
can be drawn with parametric equations. If \( x = r \cos(t) \), then

\[ r = \ldots \]

and \( y = \ldots \)

9. (W) A certain bacteria population is know to triple every 30 minutes. Suppose that there are initially 500 bacteria. What is the size of the population after \( t \) hours?

10. The point \( P(3, 18) \) lies on the curve \( y = x^2 + x + 6 \). If \( Q \) is the point \( (x, x^2 + x + 6) \), find the slope of the secant line \( PQ \) for any value of \( x \):

\[ m_{PQ} = \]

Now, evaluate the slope for the following values of \( x \). (Use a calculator.)

If \( x = 3.1 \), the slope of \( PQ \) is \( \ldots \),

and if \( x = 3.01 \), the slope of \( PQ \) is \( \ldots \),

and if \( x = 2.9 \), the slope of \( PQ \) is \( \ldots \),

and if \( x = 2.99 \), the slope of \( PQ \) is \( \ldots \).

Based on the above results, guess the slope of the tangent line to the curve at \( P(3, 18) \).

11. Let \( f(x) = \begin{cases} 2 & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ x & \text{if } x > 3 \end{cases} \)

Sketch the graph of \( f \) and evaluate the following. Write “DNE” if the limit does not exist.

\[ f(3) = \quad f(4) = \quad \lim_{x \to 3^-} f(x) = \quad \lim_{x \to 3^+} f(x) = \quad \lim_{x \to 3} f(x) = \]

12. (W) (a) Evaluate the limit.

\[ \lim_{x \to -2} \frac{x^2 - x - 6}{x + 2} \]

(b) Evaluate the limit.

\[ \lim_{t \to -5^+} \frac{|t + 5|}{t + 5} \]