MAT 136 (Calculus I) Prof. Swift
Exam 2 will be Feb. 24, 2003 on Chapter 2

No notes are allowed. A graphing calculator and a straight-edge or ruler are expected. Show your work! If you don’t show your work, or you have errors in the work, I may take off points even if you have the correct answer.

Here is the exam I gave the last time I taught this course.

1. Find the limit, or determine that the limit does not exist, without the aid of your calculator. Show your work.

   (a) \( \lim_{x \to 1} \frac{x^2 - 1}{x - 1} \)

   (b) \( \lim_{x \to \infty} \frac{x^2 - 2}{2x^2 - x + 1} \)

   (c) \( \lim_{x \to 0} \frac{|x|}{x} \)

2. (a) Estimate the following limit to 3 significant figures with the aid of your calculator, and describe what you did.

   \( \lim_{x \to 0} \frac{2^x - 1}{x} \approx \) ________.

   (b) Using the result of part (a), fill in the blanks:

   For the function \( f(x) = \) __________, \( f'(\ ) \approx \) ________.
3. Use the definition of the derivative to compute the derivative of $f(x) = x^2$. 
Hint: The definition of $f'(x)$ involves a limit.

4. The figure shows the graphs of $f$, $f'$, and $f''$. Identify each curve. (You do not need to explain your choices.)
5. Recall that the function $f(x) = \sqrt{x}$ has the derivative $f'(x) = \frac{1}{2\sqrt{x}}$.

(a) Find an equation for the tangent line to $y = \sqrt{x}$ at the point $(25, 5)$.

(b) Use your answer to part (a) to approximate $\sqrt{23}$ and $\sqrt{26}$ without the aid of your calculator.

6. For each of the functions $f$ shown, sketch the graph of $f'$ on the same axes.
7. True/False. Determine whether the statement is true or false. You do not need to justify your answers.

(a) True/False. If $f(0) = 0$ and $g(0) = 0$, then $\lim_{x \to 0} \frac{f(x)}{g(x)}$ does not exist.

(b) True/False. $\lim_{x \to 0} \sin(\pi/x) = 0$.

(c) True/False. $\lim_{x \to 0} x \sin(\pi/x) = 0$.

8. The graph of the derivative $f'$ of a function $f$ is shown.

(a) On what intervals is $f$ increasing or decreasing?

(b) At what values of $x$ does $f$ have a local maximum or minimum?

(c) On what intervals is $f$ concave upward or downward?

(d) State the $x$ coordinates of the points of inflection.

(e) Assuming that $f(0) = 0$, sketch the graph of $f$. 